The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2021-2022

MECHANICS OF SOLIDS

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

Only a calculator from approved list A may be used in this examination.

Basic Models	***Scientific Calculators***
Aurora HC133 Casio HS-5D Deli – DL1654 Sharp EL-233	Aurora AX-582 Casio FX82 family Casio FX83 family Casio FX85 family Casio FX350 family Casio FX570 family Casio FX 991 family Sharp EL-531 family Texas Instruments TI-30 family Texas BA II+ family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: MCQ answer sheet, answer book/s, formula sheet

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

SECTION A

1. If the expression of strain energy for a beam is:

$$U = \frac{a^3P^2}{8EI} + \frac{b^3P^2}{6EI} + \frac{a^2bP^2}{4GJ}$$

then what is the corresponding expression for deflection, u, at the position, and in the direction of load, P?

A. $u = \frac{P}{2}\left(\frac{a^3}{4EI} + \frac{b^3}{3EI} + \frac{a^2b}{2GJ}\right)$

B. $u = \frac{P^3}{6}\left(\frac{a^3}{4EI} + \frac{b^3}{3EI} + \frac{a^2b}{2GJ}\right)$

C. $u = P\left(\frac{a^3}{4EI} + \frac{b^3}{3EI}\right)$

D. $u = P\left(\frac{a^3}{8EI} + \frac{b^3}{6EI}\right)$

E. $u = P\left(\frac{a^3}{4EI} + \frac{b^3}{3EI} + \frac{a^2b}{2GJ}\right)$

[2]

2. For a strut of length, L, which is fixed at $x = 0$ and at $x = L$, and subjected to compressive loading, the comprehensive set of boundary conditions are:

A. (i) at $x = 0$, $y = 0$, (ii) at $x = L$, $y = 0$ & (iii) at $x = L$, $\frac{dy}{dx} = 0$

B. (i) at $x = 0$, $y = 0$, (ii) at $x = 0$, $\frac{dy}{dx} = 0$, (iii) at $x = L$, $y = 0$ & (iv) at $x = L$, $\frac{dy}{dx} = 0$

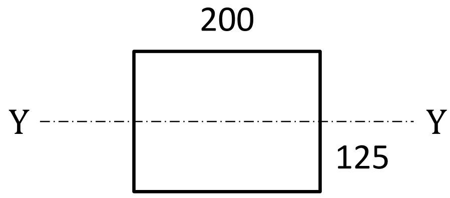
C. (i) at $x = L$, $y = 0$ & (ii) at $x = L$, $\frac{dy}{dx} = 0$

D. (i) at $x = 0$, $y = 0$ & (ii) at $x = L$, $y = 0$

E. (i) $x = 0, \frac{dy}{dx} = 0$ & (ii) $x = L, \frac{dy}{dx} = 0$

[2]

3. If a beam has a rectangular cross-section, as shown in Fig. Q3, a yield stress, $\sigma_y = 207\ \mathrm{MPa}$, and is subjected to a pure bending moment of $50\ \mathrm{kNm}$, about the $\mathrm{Y-Y}$ axis, does yielding occur?



All dimensions in mm

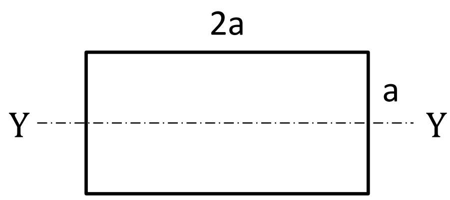
Fig. Q3

Assume elastic-perfectly-plastic material behaviour.

A. Yes
B. -
C. -
D. -
E. No

[2]

4. A beam with a rectangular cross section, that is subjected to a pure bending moment about the Y-Y axis, as shown in Figure Q4, is required to have a corresponding 2nd moment of area of at least $2.2 \times 10^7\ \mathrm{mm^\wedge 4}$. What is the minimum required value of a?



All dimensions in mm

Fig. Q4

A. 57.6 mm
B. 127.5 mm
C. 107.2 mm
D. 509.2 mm
E. 75.8 mm

[2]

5. At intermediate alternating stresses and when conservatism is required in the prediction of fatigue life:

A. The Gerber curve is a better option than the Soderberg line or the Goodman line
B. The Goodman line is a better option than the Soderberg line or the Gerber curve
C. The Soberberg line is a better option than the Goodman line or the Gerber curve
D. There is no difference in the Soderberg line, the Goodman line or the Gerber curve
E. Either of the Gerber curve or the Goodman line is the best option

[2]

6. If a beam under bending has the following 2nd order differential equation:

$$EI\frac{d^2y}{dx^2} = \frac{R_A}{2}x + \frac{M_o}{3}\langle x-2\rangle^0 - P_o\langle x-3\rangle$$

then what is the corresponding expression for deflection in the beam?

A. $y = \frac{R_A x^3}{12} + \frac{M_o\langle x-2\rangle^2}{6} - \frac{P_o\langle x-3\rangle^3}{6} + Ax + B$

B. $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{4} + \frac{M_o\langle x-2\rangle}{3} - \frac{P_o\langle x-3\rangle^2}{2} + A\right)$

C. $y = \frac{1}{EI}\left(\frac{R_A x^3}{12} + \frac{M_o\langle x-2\rangle^2}{6} - \frac{P_o\langle x-3\rangle^3}{6} + Ax + B\right)$

D. $\frac{dy}{dx} = \frac{R_A x^2}{4} + \frac{M_o\langle x-2\rangle}{3} - \frac{P_o\langle x-3\rangle^2}{2} + A$

E. $EIy = \frac{R_A x^3}{12} + \frac{M_o\langle x-2\rangle^2}{6} - \frac{P_o\langle x-3\rangle^3}{6} + Ax$

[2]

7. Fig. Q7 demonstrates which type of material behaviour?

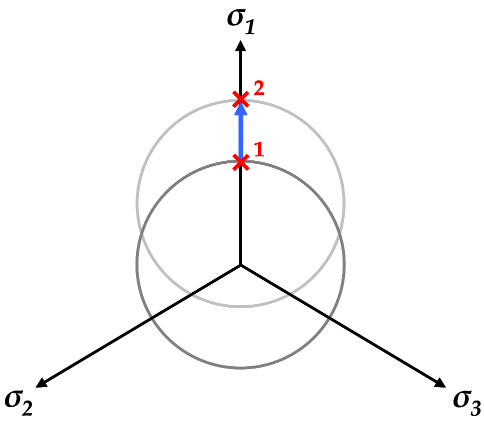


Fig. Q7

- **A.** Kinematic hardening
- **B.** Buckling
- **C.** Isotropic hardening
- **D.** Fracture
- **E.** Mixed isotropic and kinematic hardening

[2]

8. The expression for strain energy in a beam under torsion is:

A. $U = \int_0^L \frac{P^2}{2AE} dx$

B. $U = \int_0^L \frac{M^2}{2EI} dx$

C. $u = \frac{\partial U}{\partial P}$

D. $U = \int_0^L \frac{T^2}{2JG} dx$

E. $u = \int_0^L \frac{T^2}{2EI} dx$

where P is load, M is bending moment and T is torque.

[2]

9. For a steel column of rectangular cross-section, $40\ \text{mm} \times 20\ \text{mm}$, and of length $2.75\ \text{m}$, that is fixed at both ends and has a Young's modulus, E, of $210\ \text{GPa}$, the critical buckling load is:

A. 7.3 kN
B. 29.2 kN
C. 14.9 kN
D. 116.9 kN
E. 1.8 kN

[2]

10. Fig. Q10 illustrates which type of material behaviour?

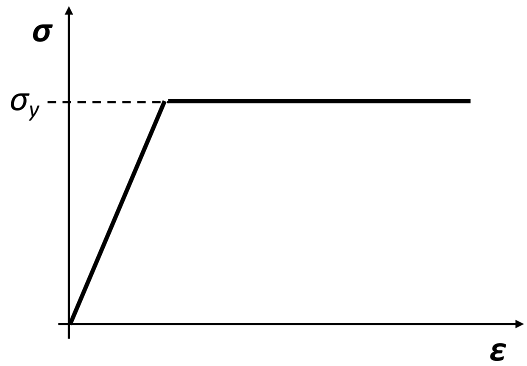


Fig. Q10

A. linear softening
B. elastic-perfectly-plastic
C. non-linear elasticity
D. non-linear hardening
E. linear hardening

[2]

11. If a material expands freely due to heating it will develop

A. thermal stress
B. tensile stress
C. bending
D. no stress
E. compressive stress

[2]

12. Maximum shear stress theory is very commonly used for

A. brittle material
B. ductile material
C. both ductile as well as brittle materials
D. compressive loading only
E. none of the above

[2]

13. A closed-ended thin-walled cylinder is filled with gas. The inner diameter of the cylinder is 200 mm and the wall thickness is 2 mm. What is the maximum permissible gas pressure in the cylinder if the allowable tensile and shear stresses in the cylinder are 7 MPa and 2 MPa, respectively?

A. 140 kPa
B. 160 kPa
C. 180 kPa
D. 200 kPa
E. 220 kPa

[2]

14. For the same thin-walled cylinder in Q13, if the internal gas pressure is 100 kPa, what is the Tresca equivalent stress on the outer surface?

A. 1.0 MPa
B. 2.5 MPa
C. 5.0 MPa
D. 7.5 MPa
E. 10.0 MPa

[2]

15. When a body is subjected to a direct tensile stress in one plane, the maximum shear stress is _______ the maximum normal stress.

A. equal to
B. one-third of
C. one-half of
D. two-thirds of
E. twice

[2]

16. A 600 mm long, 1D bar element has a rectangular cross section of 40 mm × 50 mm and is made of a material with a Young's Modulus of 70 GPa. The stiffness matrix of the element (in N/m) is:

A. $\begin{bmatrix} 70 \times 10^9 & 0 \\ 0 & 70 \times 10^9 \end{bmatrix}$

B. $\begin{bmatrix} 70 \times 10^9 & -70 \times 10^9 \\ -70 \times 10^9 & 70 \times 10^9 \end{bmatrix}$

C. $1.40 \times 10^8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $1.40 \times 10^8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

E. $2.33 \times 10^8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

[2]

17. For the beam cross-section shown in Figure Q17, the shear centre will be located closest to which point?

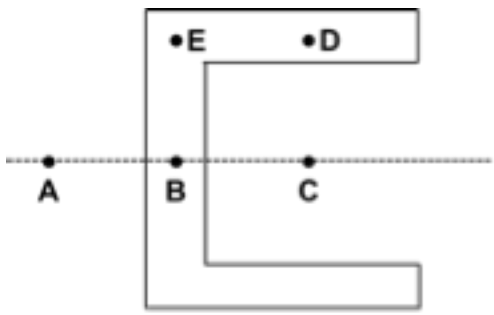


Fig. Q17

A. A
B. B
C. C
D. D
E. E

[2]

18. A bar with a rectangular cross-section ($b \times d$) of 40 mm × 50 mm is subjected to a vertical shear force of 25 kN. What is the value of maximum shear stress in the bar?

A. 18.75 MPa
B. 37.50 MPa
C. 75.00 MPa
D. 150.00 MPa
E. 9.38 MPa

[2]

19. A rotor disc with an angular velocity of 3000 rpm has an external diameter of 0.9 m and has a 0.2 m diameter hole bored along its axis. What is the value of hoop stress at the external diameter? (ρ = 7200 $\frac{kg}{m^3}$, ν = 0.25).

A. 16.4 MPa
B. -16.4 MPa
C. 32.8 MPa
D. -32.8 MPa
E. 0 MPa

[2]

20. In the case of pure shear at a point, the sum of the normal stresses on two orthogonal planes is equal to:

A. zero
B. the maximum shear stress
C. twice the maximum shear stress
D. the maximum tensile stress
E. the maximum compressive stress

[2]

SECTION B

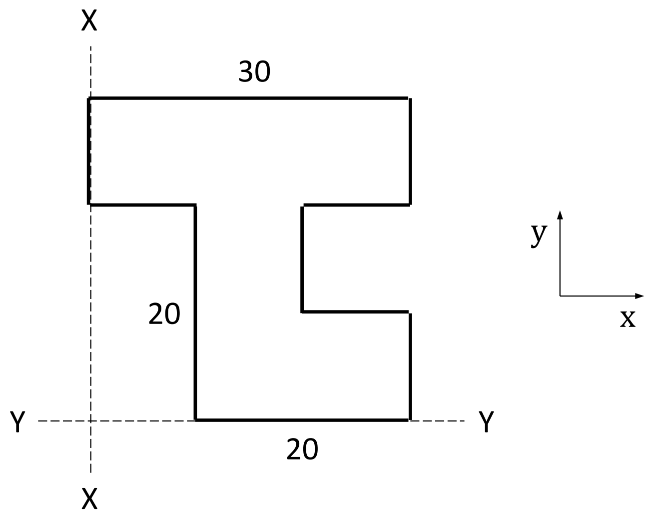
21. (a) An open-ended thick-walled cylinder of internal diameter 0.2 m is supported such that there is no longitudinal stress. The cylinder is subjected to an internal pressure of p_i = 80 MPa. If the maximum direct stress is to be limited to 250 MPa, determine the minimum possible wall thickness required.

[12]

(b) With this calculated wall thickness, determine the increase in the external diameter, due to expansion, when the pressure is applied. Assume E = 70 GPa.

[8]

22. Figure Q22 shows the cross-section of a beam. All unlabelled edges are 10 mm.



All dimensions in mm

Fig. Q22

Determine:

(a) The position of the Centroid of Area, C. [5]

(b) The principal 2nd Moments of Area. [10]

(c) The orientation of the Principal Axes with respect to the x-y co-ordinate system (show on a sketch of the cross-section). [5]

23. Three rigid bodies (represented as 2, 3 and 4) and a rigid wall (represented as 1) are connected by four springs (represented as A, B, C and D), as shown in Figure Q23. The stiffness of the four springs (in N/mm) are k_A = 300, k_B = 400, k_C = 500 and k_D = 600, respectively. A horizontal force of 1,500 N (represented as F_4) is applied on body 4 as shown in the figure. Assuming that the bodies can undergo translation in the horizontal direction only, use the finite element method to find the displacements of the three rigid bodies and the forces in the four springs, assuming linear elastic behaviour. State whether the forces are tensile or compressive.

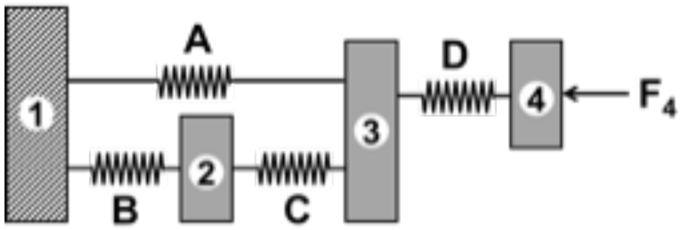


Fig. Q23

[20]